D II Physics Hons (Paper - IV)

Dr Sanjeet Kumar, Asst. Prof. - Deptt of Physics

[illegible], Arrah, Bihar

India.

Electrostatics

Molecular Polarisability:

When a dielectric is placed in an electric field, its molecules become electric dipoles oriented in the direction of the field. Thus the dielectric acquires a net dipole moment and its molecules are said to be 'polarised'.

For most molecules the dipole moment is proportional to the electric field causing polarisation. The dipole moment of a molecule per unit polarising field is called 'polarisability' of the molecule.

(*) Molecular Field in a Dielectric:

Fig 1

The electric field which is responsible for polarising a molecule of the dielectric is called the 'molecular field' or 'polarising field'. If the is a gas (whose molecules are at large distances from one another).

the polarising field is simply the externally applied field (which is known as macroscopic electric field). In case of solid or liquid dielectrics, however, the actual field acting on a molecule of the dielectric is different from the external field. It includes not only the external field but also the field produced by all polarised molecules (dipoles) in the dielectric with the exception of the molecule under consideration (because it will not be polarised by its own field). This is known as the 'local' electric field acting on a molecule and is responsible for the polarisation of this particular molecule.

The local field at a particular position can be calculated in the following way. Let the dielectric material be placed in the uniform electric field between two parallel plates of a capacitor (Fig. 1 a). Let us suppose we want to compute the field at position A of a molecule. We may assume that this molecule is not present at all. We draw a sphere around A of such size that we may consider all molecules outside the sphere to be far away. In other words, the dielectric outside the sphere may be treated as a continuum of dipoles (macroscopic point of view). The molecules inside the sphere are, however, to be

treated as individual dipoles (not as a continuum).
Now, the local field at A is due to three sources:
(a) Free charges on the capacitor plates (externally applied field).

(b) Polarised molecules (dipoles) of the dielectric outside the sphere. To evaluate this contribution, the dielectric material outside the sphere may be replaced by bound (induced) charges on the outer faces of the dielectric and also on the surface of the sphere, as shown in fig. 1b.
(c) Polarised molecules within the sphere.
The local electric field at A may be written as

$$E_{local} = E_0 + (E_1 + E_2) + E_3 . \quad \text{---(i)}$$

E_0 is the external field due to the charged plates of the capacitor and is given by

$$E_0 = \frac{\sigma}{\epsilon_0},$$

where σ is the free charge density on the capacitor plates.
E_1 is the depolarising field due to the bound charges on the outer faces of the dielectric and is given by

$$E_1 = -\frac{\sigma'}{\epsilon_0},$$

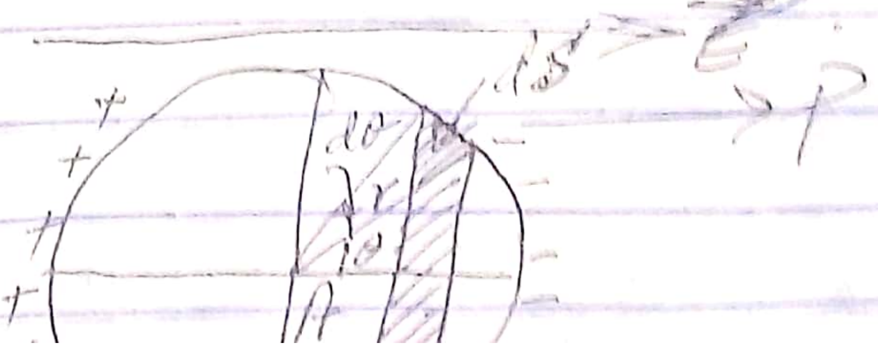
where σ' is the bound charge density.
(The negative sign signifies that E_1 opposes E_0).
E_2 is the field due to the bound charges on the surface of the sphere.

As we know that the net macroscopic electric field within the dielectric is given by:

$$E = \frac{\sigma}{\epsilon_0} - \frac{\sigma'}{\epsilon_0},$$

so that we have

$$E_{local} = E + E_2 . \quad \text{(ii)}$$



over dS is therefore $P\cos\theta$. The charge on dS is $P\cos\theta \, dS$, and the field at A due to this charge is $\frac{1}{4\pi\epsilon_0}\frac{P\cos\theta \, dS}{r^2}$, where r is the radius of the sphere.

The field will be represented by a vector directed from A to dS, of which the component in the direction of $\vec{E}$ is

$$\left(\frac{1}{4\pi\epsilon_0}\frac{P\cos\theta \, dS}{r^2}\right)\cos\theta = \frac{P\cos^2\theta \, dS}{4\pi\epsilon_0 r^2}.$$

Now, suppose that dS is a ring-shaped element (shown shaded) of radius $r\sin\theta$, and width $r\,d\theta$, on the surface of the sphere. The area of this element is;

$$dS = 2\pi(r\sin\theta)\,r\,d\theta = 2\pi r^2\sin\theta\,d\theta.$$

The component of the field at A perpendicular to $\vec{E}$ due to this ring is zero, since such components are symmetrically distributed around the axis. The component of field along the direction of $\vec{E}$ is

$$\frac{P\cos^2\theta \, dS}{4\pi\epsilon_0 r^2} = \frac{P\cos^2\theta(2\pi r^2\sin\theta\,d\theta)}{4\pi\epsilon_0 r^2}$$

$$= \frac{P}{2\epsilon_0}\cos^2\theta\sin\theta\,d\theta$$

The field (E_2) at A due to the entire induced charge on the surface of the sphere is

$$E_2 = 2\int_0^{\pi/2} \frac{P}{2\epsilon_0} \cos^2\theta \sin\theta \, d\theta$$

$$= \frac{P}{\epsilon_0} \left| -\frac{\cos^3\theta}{3} \right|_0^{\pi/2} = \frac{P}{3\epsilon_0}.$$

Let us now put this value of E in eqn. (ii)

$$\boxed{E_{local} = E + \frac{P}{3\epsilon_0}} \quad \text{- - - (iii)}$$

This is the actual field at the position of a molecule within the dielectric.